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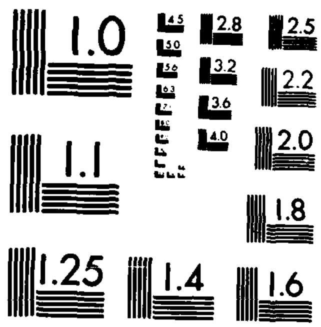

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Principles of Magnetospheric Ion Composition

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17 December 1984

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Ion composition provides an important diagnostic tool for identifying and quantifying source, loss, energization, and transport processes applicable to magnetospheric charged particles. However, the situation is not as simple as had been envisioned a decade ago. Ring-current and radiation-belt ions are believed to come from either the sun or the earth's ionosphere or both. Entry is not direct in either case. Solar-wind velocity and ion composition vary considerably with time. Ion-velocity distributions are partially thermalized

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in the magnetosheath after crossing the bow shock. Entry of solar-wind ions into the magnetosphere is modulated in part by reconnection efficiency, i.e., by the ratio of cross-tail electric field to "asymptotic" interplanetary electric field. Non-adiabatic motion in the plasma sheet may enable the cross-tail electric field to heat the various ion species by different amounts. Entry of ionospheric ions into the plasma sheet seems to occur primarily through afternoon- and evening-sector auroral arcs, rather than through the polar wind as had previously been postulated. Energization occurs through interaction with the auroral potential structure (which varies with magnetic activity) and through plasma-sheet processes cited above. These are some of the uncertainties that affect the outer boundary condition for the problem of magnetospheric ion composition. Radial transport of magnetospheric ions at ring-current energies ($E \sim 10$-100 keV) is achieved primarily by unsteady electrostatic convection. The corresponding diffusion coefficient is independent of charge and mass among particles having drift periods $\gg 2\pi$ times the postulated decay time of an impulse in the convection electric field. Erosion of the ring current occurs primarily through charge exchange and perhaps through wave-particle interactions. Charge exchange favors the survival of O^+ and He^+ over H^+ and He^{++} at energies < 50 keV, but account must be taken of population disparities in the source region and of time scales associated with radial diffusion therefrom. Collisional processes are less important at radiation belt energies ($E > 200$ keV), and the mapping of ion distributions from the source region to low L values can be partially understood in terms of a common diffusion profile with M and J (first two adiabatic invariants) conserved. The pitch-angle anisotropy of ring-current ions is likely to generate electrostatic ion-cyclotron waves outside the plasmasphere and electromagnetic ion cyclotron waves inside the plasmasphere. These should result in preferential pitch-angle diffusion of the major ionic constituent and in a possibly weaker (parasitic) pitch-angle diffusion of the minor ionic constituents.

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CONTENTS

FIGURES

INTRODUCTION

In more innocent times it was believed that the ion composition of the ring current and radiation belts at a specified energy/nucleon should be the same as the ion composition of the solar wind. The idea was that traversal of the bow shock would separately thermalize each ionic constituent of the solar wind so as to produce a Maxwellian distribution having a temperature ~ 1 keV/nucleon as an outer boundary condition at L ~ 10 on the phase-space density, and that the diffusion equation would impose a common profile (defined by the requirement of a divergence-free radial-diffusion current) on all the constituents in the ring current and radiation belts. This concept has largely failed to be supported by the observational data and so is considered obsolete.

One difficulty with the above concept is that the *e*-folding energy provided by thermalizing the solar wind is too small by an order of magnitude (see below) for assignment to a boundary at L ~ 10. Another difficulty is the neglect of ionospheric ions, which can enter the magnetosphere with several keV of energy by virtue of the field-aligned potential drop characteristic of auroral arcs in the afternoon and evening sectors. It seems, however, that entry into the radiation belt or ring current is not direct in either case, but rather occurs via the plasma sheet. Violation of the first adiabatic invariant of charged-particle motion within the plasma sheet may enable the cross-tail electric field to heat the various ionic species further, but not necessarily by the same amount or by the same amount per nucleon. Moreover, entry of solar wind ions into the plasma sheet is modulated in part by reconnection efficiency, i.e., by the ratio of cross-tail electric field to asymptotic interplanetary electric field. The reconnection efficiency can vary considerably with time, as can the solar-wind velocity and ion-composition.

Likewise, the field-aligned auroral potential drop can vary considerably with time. These are some of the uncertainties that can affect the outer boundary condition for the problem of magnetospheric ion composition as a function of energy/nucleon.

Radial transport of magnetospheric ions at ring-current energies ($E \sim 10 - 100$ keV) is achieved primarily by unsteady electrostatic convection. The corresponding diffusion coefficient is independent of charge and mass among particles having drift periods >> 2 hr if twenty minutes is the postulated decay time of an impulse in the convection electric field. However, the diffusion current is not divergence-free, since distributed losses constitute an essential ingredient of ring-current dynamics. Erosion of the ring current occurs primarily through charge exchange and perhaps through wave-particle interactions. Charge exchange favors the survival of O^+ and He^+ over H^+ and He^{++} at energies ≤ 50 keV, but account must be taken of population disparities in the plasma sheet and of time scales associated with radial diffusion therefrom. Thus, it is possible for H^+ to predominate over O^+ and He^+ in the early stages of ring-current formation but not in the later stages of a magnetic storm.

Collisional processes are less important at radiation-belt energies ($E \geq 200$ keV), and the mapping of ion distributions from the outer boundary to low L values can be partially understood in terms of a common diffusion profile with M and J (first two adiabatic invariants) conserved. One consequence of this is the appearance of very large ionic pitch-angle anisotropies in the inner radiation zone. Indeed, the pitch-angle anisotropy at low L values is a reflection of the energy spectrum at the outer-boundary location. Inner-zone observations typically reveal that the alpha-particle anisotropy is even larger than the proton anisotropy at the same energy/nucleon. This means that the α/p (alpha/proton) intensity ratio must vary inversely with magnetic latitude along a field line, as indeed it does. The occurrence of a larger alpha-particle than proton anisotropy at fixed energy/nucleon can be understood in terms of an alpha-particle temperature that is less than four times the proton temperature at the outer boundary of trapped radiation.

Much of the observational research emphasis on ion composition in recent years has focused on energies ≤ 20 keV, i.e., on energies so low that they are not truly in the radiation-belt range. Indeed, such energies are so low that they hardly include a significant portion of even the ring-current spectrum. It is therefore quite important to avoid sweeping generalizations in the interpretation of ion-composition results. Meanwhile, progress (e.g., Gloeckler, 1977) is being made toward the construction of ion-identifying spectrometers that operate in what had been the energy "gap" between ~ 20 keV/charge and ~ 100 keV/nucleon, as required for proper scrutiny of the ring-current spectrum.

The purpose of this work is to outline the basic principles upon which the theoretical study of magnetospheric ion composition is founded. The author helped to compile a much more thorough review of the literature on this subject only a few years ago (Cornwall and Schulz, 1979). Not much would be gained by providing the results of yet another literature search at this time. Thus, the focus of the present review is directed instead on the enduring fundamentals. These relate to the problems of access, energization, transport, and loss of protons and other ions with respect to the earth's magnetosphere and radiation belts.

ACCESS

The main sources of magnetospheric ions are the galaxy, the sun, and the ionosphere. The galaxy provides cosmic-ray ions that are themselves too energetic to be trapped by the earth's magnetic field. However, the backscatter (albedo) products of spallation reactions that occur when such cosmic-ray ions strike the upper atmosphere include many neutrons in the 10-100 MeV range. The occasional decay of a neutron from this population contributes a proton in the 10-100 MeV energy range and an electron of energy < 800 keV to the radiation belts. Other sources of electrons are much more impor tha this one. Other sources of inner-zone ions in the 10-100 MeV energy range are negligible. Thus, inner-belt ion composition is not much of a problem at these very high energies: the ions in this range are almost exlusively protons. The source described in this paragraph is known by the acronym CRAND (cosmic-ray-albedo-neutron decay).

Drift shells at synchronous altitude and beyond are occasionally populated by solar-flare ions (presumably protons) of energy 5-70 MeV (Paulikas and Blake, 1969). The mode of access in this case is largely direct, in that the drift shells populated are those that pass within a gyro-radius of the magnetopause or neutral sheet.

Of greater interest in the context of magnetospheric ion composition are particles of much lower energy, perhaps $E \sim 1$-10 keV, for which the modes of access are largely indirect. These include magnetosheath ions derived from the shocked solar wind and auroral ions energized by the electric potential drop that occurs along magnetic field lines in the PM (afternoon-evening) sector of the auroral oval. Magnetosheath ions with $E \sim 1$ keV might impinge on the magnetosphere anywhere, but the convection electric field tends to expel them from the dayside region of closed field lines. Conversely, charged particles that enter at the polar cleft (a dayside region of weak magnetic field surrounding the neutral points) are drawn into the tail lobes by the convection electric field, as are particles that impinge on the tail-lobe surfaces. Characteristics of the resulting plasma mantle have been described by Sckopke and Paschmann (1978).

The convection velocity of plasma in either tail lobe is smaller than the solar-wind velocity by a factor of order $\epsilon\bar{B}_z/B_t \ll 1$, where B_t is the tail field, $\bar{B}_z$ is the southward component of the interplanetary magnetic field, and ϵ is the "reconnection efficiency" defined by Kennel and Coroniti (1975) as the ratio of the east-west component of the cross-trail electric field to that of the interplanetary electric field. Observations of Burton <u>et al.</u> (1975) suggest a "half-wave rectifier" model such that ϵ is practically zero for northward $\bar{B}_z$ but at least 0.2 (and perhaps even ~ 1) for southward $\bar{B}_z$. The factor $\epsilon\bar{B}_z/B_t$ is quite small (≤ 0.1) in any case, and so only the extreme flanks of the plasma mantle (i.e., regions already within a few earth radii of the current sheet in the tail) are likely to populate the plasma sheet by convection within the first few tens of earth radii downstream from the earth. The reversal of magnetic-field direction across the current sheet and the dawn-to-dusk direction of the cross-tail electric field make the low-latitude AM (pre-dawn) flank an inviting place for magnetosheath ions to enter the magnetosphere in any case.

The access of ionospheric ions to the plasma sheet is made possible by the electrostatic potential structure characteristic of the PM (afternoon-evening) sector of the auroral oval. It is in this region (near the boundary between closed and open magnetic field lines) that there develops a virtual discontinuity in the magnetospheric convection electric field if one treats magnetic field lines as electrostatic equipotentials. There is a Pedersen conductance owing to the collisionality of the ionosphere and an Ohmic impedance due to the mirror forces exerted by the inhomogeneous magnetic field on the hot auroral plasma. It seems that these properties conspire with the requirement of current conservation (Ampère's law) to produce a multi-kilovolt potential drop along the field lines of the auroral oval. The geometry is such that the corresponding parallel (to $\underset{\sim}{B}$) component of the electric field should be upward in the PM sector and downward (if present) in the AM sector. Statistics compiled by Ghielmetti et al. (1978) from S3-3 data suggest that upgoing ion beams are about six times more probable in the PM sector than in the AM sector, and about 44 times more probable in the dusk quadrant (15-21 hr, magnetic local time) than in the dawn quadrant (03-09 hr, MLT).

The contribution of auroral ion beams to the plasma sheet presumably exceeds that of the polar wind, which had been until recently the favored medium (e.g., Axford, 1970) for supplying ionospheric ions to the plasma sheet. The polar wind theoretically consists of protons (along with a few helium ions) having energies ≤ 10 eV. The auroral ion beams seem to be rich in oxygen as well, with ion energies of several keV at least. It is true that the polar wind delivers ions from the entire polar cap rather than from only the PM half of the auroral oval, but the deficiencies in ion energies and oxygen ions now seem to make the polar wind a less important medium (than the auroral ion beams) for the transfer of ionospheric ions to the plasma sheet. Thus, it seems that the main sources of 1-10 keV ions for the plasma sheet are (1) magnetosheath plasma derived from the shocked solar wind and (2) ionospheric plasma from the PM sector of the auroral oval.

ENERGIZATION

The importance of the plasma sheet for magnetospheric ion composition resides in the adiabatic and non-adiabatic processes by which ions are further energized there before being transported into the region of closed drift shells. The adiabatic energization process entails the usual gradient-curvature drift of nightside ions across equipotentials of the dawn-to-dusk electric field, or (equivalently) the sunward convection of nightside ions into a spatially increasing magnetic-mirror field. This process occurs in the earthward portion of the nightside plasma sheet, i.e., in a region where the magnetic field lines are topologically dipolar but from which the adiabatic drift shells intersect the magnetopause. The particles in question are "quasi-trapped" in the terminology of Roederer (1970), but the failure of their drift shells to close is as much the fault of the convection electric field as it is of the gradient-curvature drift.

The non-adiabatic energization process occurs in the tailward portion of the plasma sheet, where the northward ($\hat{\underset{\sim}{z}}$) component of the earth's magnetic field is too weak to enforce conservation of the first two adiabatic invariants. A plasma physicist's prototype for the magnetic field in this distinctly non-dipolar region might be expressed as

$$\underset{\sim}{B} = \hat{\underset{\sim}{z}}\, B_z - \hat{\underset{\sim}{x}}\, B_t \tanh (z/z_0), \tag{1}$$

where $\hat{\underset{\sim}{x}}$ is the anti-sunward unit vector, z_0 is the "half-width" of the current sheet, and $B_t \gg B_z$. A model electric field consistent with (1) is the uniform field $\underset{\sim}{E} = -\hat{\underset{\sim}{y}}|E_y|$. A theorem of Stern and Palmadesso (1975) asserts that there is no net gradient-curvature drift in a magnetic field such as (1) if

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B_z, B_t, and z_0 are constants. The proof follows from the construction of a bounce-averaged Hamiltonian function that depends on the first two adiabatic invariants M and J but not on the coordinates x_0 and y_0 at which the guiding field line crosses the plane z = 0. An immediate corollary is that the electric field $\underset{\sim}{E} = -\underset{\sim}{\hat{y}}|E_y|$ produces a bounce-averaged drift such that $\dot{x}_0 = -(c/B_z)|E_y|$, but that particle energization does not occur.

The theorem of Stern and Palmadesso (1975) clearly does not apply if B_z = 0. In this case there is a gradient drift in the + $\underset{\sim}{\hat{y}}$ direction for ions adiabatic with respect to M, but no curvature drift since the field lines are rectilinear. There is also a net convection in the $\underset{\sim}{\hat{z}}$ direction (toward the neutral sheet) for all ions and a net drift in the − $\underset{\sim}{\hat{y}}$ direction for ions that fail to conserve M because their trajectories cross the plane z = 0. Moreover, the ions that thus fail to conserve M are energized by the uniform electric field. The theorem fails in the case B_z = 0 because the second adiabatic invariant J no longer exists, or (equivalently) because the bounce average is no longer defined, even for particles that conserve the first invariant M.

The theorem of Stern and Palmadesso (1975) likewise fails even for $B_z \neq 0$ if the particle of interest has too large a rigidity to conserve M and J in the field configuration given by (1). Thus, the dawn-to-dusk electric field can be expected to add extra energy to ions that have entered the plasma sheet with rigidity $p/q \gtrsim z_0 B_z/c$. Recent results of Lyons and Speiser (1982) suggest that the energy gained in this way is indeed an increasing function of initial ion energy. This is a geophysical analogy of the rich becoming richer. Moreover, since the energization mechanism involves only single-particle motion in crossed electric and magnetic fields, there is no expectation that any of the energy gained by the favored ions of high initial energy will "trickle down" to the less favored ions of low initial energy.

The above-described idea for charged-particle energization in the tailward portion of the plasma sheet was first worked out in quantitative detail by Speiser (1965). It is quite remarkable that such a perceptive work has received so little attention over the past seventeen years. In any event, the occurrence of an "ion-heating" process based on single-particle motion makes it premature to invoke plasma turbulence or reconnection as the energization mechanism of the plasma sheet. It appears from the work of Lyons and Speiser (1982) that non-adiabatic single-particle motion in the crossed electric and magnetic fields is sufficient to account for ion temperatures observed in the tailward portion of the plasma sheet. Still open is the important question of whether the various ionic species are "heated" by different amounts (per particle, per unit charge, or per nucleon) by this mechanism. Sunward convection by the dawn-to-dusk electric field delivers the ions thus energized into the earthward portion of the plasma sheet, where further energization occurs through the conservation of M and J in the crossed inhomogeneous magnetic and electric fields that prevail there. Of course, a charged particle can gain even more energy if it is transported (e.g., by guiding-center diffusion) across the boundary between open (plasma-sheet) and closed (ring-current) drift shells, and from there to progressively smaller L values within the region of geomagnetically trapped particles. However, the subject of charged-particle energization by guiding-center diffusion is included below in the section about charged-particle transport across adiabatic drift shells.

TRANSPORT

For charged particles that conserve their first two adiabatic invariants M and J, the problem of non-adiabatic transport typically entails violation of the third invariant Φ through radial diffusion. At radiation-belt energies it is usual to adopt the dimensionless label L (roughly speaking, the equatorial radius of the drift shell in earth radii) as diffusion coordinate, and this can be done by means of the transformation $L \equiv 2\pi(\mu/a)|\Phi|^{-1}$, where μ is the magnetic moment of the earth and a is the radius of the earth. The diffusion equation for the drift-averaged phase-space density $\bar{f}$ then takes on the form

$$\frac{\partial \bar{f}}{\partial t} = L^2 \frac{\partial}{\partial L}\left[\frac{D_{LL}}{L^2}\frac{\partial \bar{f}}{\partial L}\right] \tag{2}$$

where D_{LL} is the diffusion coefficient (e.g., Schulz and Lanzerotti, 1974). The functional form of D_{LL} is necessarily model-dependent, but the usual treatment yields results of the form $D_{LL} = D_{LL}^{(m)} + D_{LL}^{(e)}$, where

$$D_{LL}^{(m)} \approx 7 \times 10^{-9} L^{10} [Q(y)/180\, D(y)]^2 \ \mathrm{day}^{-1} \tag{3a}$$

represents the contribution from step-like impulses in the magnetospheric $\underline{B}$ field and

$$D_{LL}^{(e)} \approx 1 \times 10^{-10} L^{10} (\gamma Z M_0 y^2/M)^2 [T(y)/2D(y)]^2 [1 + (\Omega_3 \tau)^{-2}]^{-1} \ \mathrm{day}^{-1} \tag{3b}$$

represents the contribution from exponentially decaying impulses in the electrostatic field that drives magnetospheric convection. The auxiliary functions Q(y), D(y), and T(y) are well approximated (Schulz and Lanzerotti, 1974; Davidson, 1976) by the algebraic forms

$$Q(y) \approx -27.12667 - 45.39913\,y^4 + 5.88256\,y^8 \tag{4a}$$

$$D(y) \approx 0.4600577 + 0.1066154\,y^{3/4} - 0.1997662\,y \tag{4b}$$

and

$$T(y) \approx 1.3801730 - 0.6396925\,y^{3/4} \tag{4c}$$

where y ($= \sin\alpha_0$) is the sine of the equatorial pitch angle. Other symbols appearing in (3) include the ratio γ of relativistic mass to rest mass, the ionic charge-state number Z, the scale factor $M_0 \equiv 1$ GeV/gauss, the ionic drift frequency $\Omega_3/2\pi$, and the decay time τ ($\sim$ 1200 sec) of a model electrostatic impulse. The factor $[1 + (\Omega_3\tau)^{-2}]$ differs by at most 2.5% from unity for ions having drift periods $2\pi/\Omega_3 \le 20$ min, i.e., for ions of kinetic energy $E \ge 1.2\,Z/L$ MeV, and so the factor $[1 + (\Omega_3\tau)^{-2}]$ in (3b) may be regarded as essentially constant for such particles.

Figures 1a and 1b show the variation of $D_{LL}^{(m)}$ and $D_{LL}^{(e)}$ with L in the limit $\Omega_3\tau \gg 1$ for nonrelativistic ions having the same energy (and selected values of y) at $L = 7$. The reader may be surprised to see that D_{LL} scales as L^{10} only for $y = 1$. For $\alpha_0 \ne 90^\circ$ account must be taken of the variation of y with L at fixed M and J. According to Chen and Stern (1975) this variation is well approximated by the algebraic form

$$y^{-2} \approx 1 + 1.35048\,X - 0.030425\,X^{4/3} + 0.10066\,X^{5/3} + (X/2.760346)^2 \tag{5}$$

where $X^2 \equiv (La/\mu)(J^2/8m_0M)$ for a particle of rest mass m_0. The

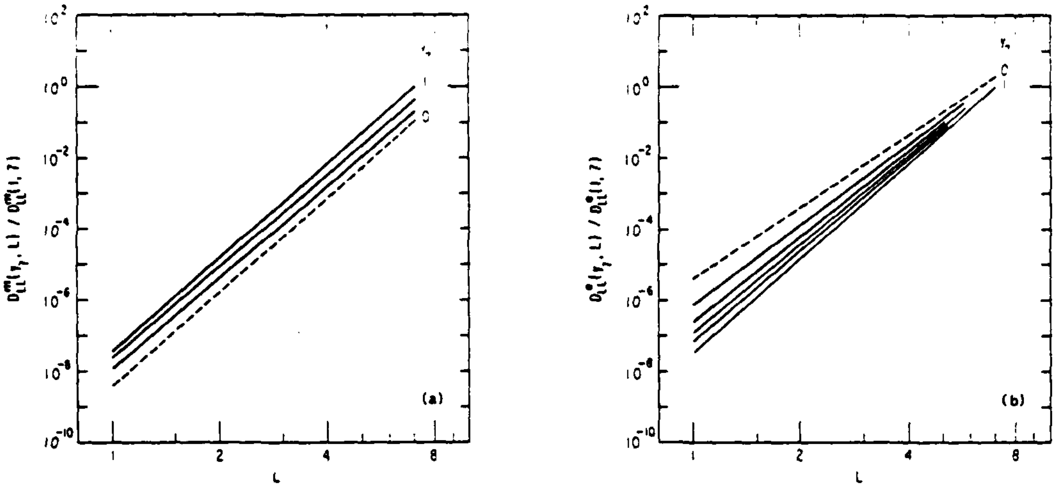

Figure 1. Radial variation of D_{LL} driven (a) by step-like magnetic impulses and (b) by electrostatic impulses that rise quickly and decay slowly on the time scale of an ion's drift period. Impulses are presumed to conserve M and J so that y varies in accordance with (5) as a function of L for each value of y_7 (denoting the value of y at L = 7), i.e, for y_7 = 0, 0.4, 0.6, and 1.0 in (a) and for y_7 = 0, 0.2, 0.4, 0.6, 0.8, and 1.0 in (b). Dashed lines (y_7 = 0) are not realized in practice, because of the loss cone (Schulz and Lanzerotti, 1974).

limiting case $y = 0$ ($X = \infty$) thus yields $y^2 \propto 1/L$ in (3b), from which it follows that $D_{LL}^{(e)}$ scales only as L^8 in this limit (see Figure 1b, dashed line). On the other hand, it seems from Figure 1a that the deviation of $D_{LL}^{(m)}$ from an L^{10} scaling is almost negligible. Thus, for ion energies sufficiently high that charge exchange and Coulomb drag can truly be neglected as they are in writing (2), one might anticipate a steady-state diffusion profile of the form

$$\bar{f}(M, J, L) = \left[\frac{1 - (L_1/L)^7}{1 - (L_1/L^*)^7}\right] \bar{f}(M, J, L^*) \tag{6}$$

where L^* (~ 10) represents the outermost closed drift shell and L_1 represents the innermost drift shell that can trap particles of a given $K^2 \equiv J^2/8m_0M$. Thus, the entire energy spectrum of $\bar{f}(M, J, L)$ at $L < L^*$ is determined by the boundary-value spectrum contained in $\bar{f}(M, J, L^*)$. Of particular interest in the case of nonrelativistic ions is the specification

$$\bar{f}(M, J, L^*) = [(y^*)^2(L^*a)^3E_0^*/\mu M]^{\ell+1}\, \bar{f}^* \exp[-\mu M/(y^*)^2(L^*a)^3E_0^*] \tag{7}$$

where ℓ, $\bar{f}^*$, and E_0^* are constants. The special case $\ell = 0$ corresponds to an exponential spectrum, since the particle flux (per unit energy and solid angle) is equal to $2m_0E\bar{f}$. The substitution $\mu M = L^3a^3y^2E$ in (7) is valid for all L values in the dipolar field model and yields

$$\bar{f}(M, J, L^*) = [(y^*/y)^2(L^*/L)^3(E_0^*/E)]^{\ell+1}\, \bar{f}^* \times \exp[-(y/y^*)^2(L/L^*)^3(E/E_0^*)] \tag{8}$$

which can be written more compactly as

$$\bar{f}(M, J, L^*) = (E_0/E)^{\ell+1}\,\bar{f}^*\exp(-E/E_0) \tag{9a}$$

by introducing the parameter

$$E_0 = (y^*/y)^2(L^*/L)^3E_0^* \tag{9b}$$

It follows from (9) that the index ℓ of a power-law spectrum remains invariant through the diffusion profile given by (6), whereas the e-folding energy E_0 of an exponential spectrum varies with L and y^* in the same manner as the energy of an individual particle for which M and J are conserved. Moreover, the variation of E_0 with L is stronger for $y^* \sim 1$ than for $y^* \ll 1$, and this property creates anisotropy in the pitch-angle distribution observed at fixed E, over and above the anisotropy contained in the factor $[1 - (L_1/L)^7]/[1 - (L_1/L^*)^7]$, at $L < L^*$.

Application of the foregoing ideas to a body of observational data is illustrated in Figure 2 (Dungey et al., 1965). The solid curves were constructed by measuring the e-folding energies E_0 of exponential proton spectra (Davis and Williamson, 1963) at equatorial pitch angles corresponding [via (5)] to $\alpha_0 = 10°, 20°, 30°$, and $90°$ at L = 7. The dashed curves show the corresponding energy variation of an individual proton but are positioned on the logarithmic ordinate so as to best fit the observed variation of E_0 with L. The dashed curves seem to converge on a common (isotropic) value of $E_0 \sim 12$ keV at $L \sim 10$. It is natural, therefore, to associate the parameters $E_0^* \sim 12$ keV and $L^* \sim 10$ with the outer boundary condition on the proton radiation belt, on which the observations of Davis and Williamson (1963) spanned the energy range $E \approx 0.1$-1.6 MeV. These results for

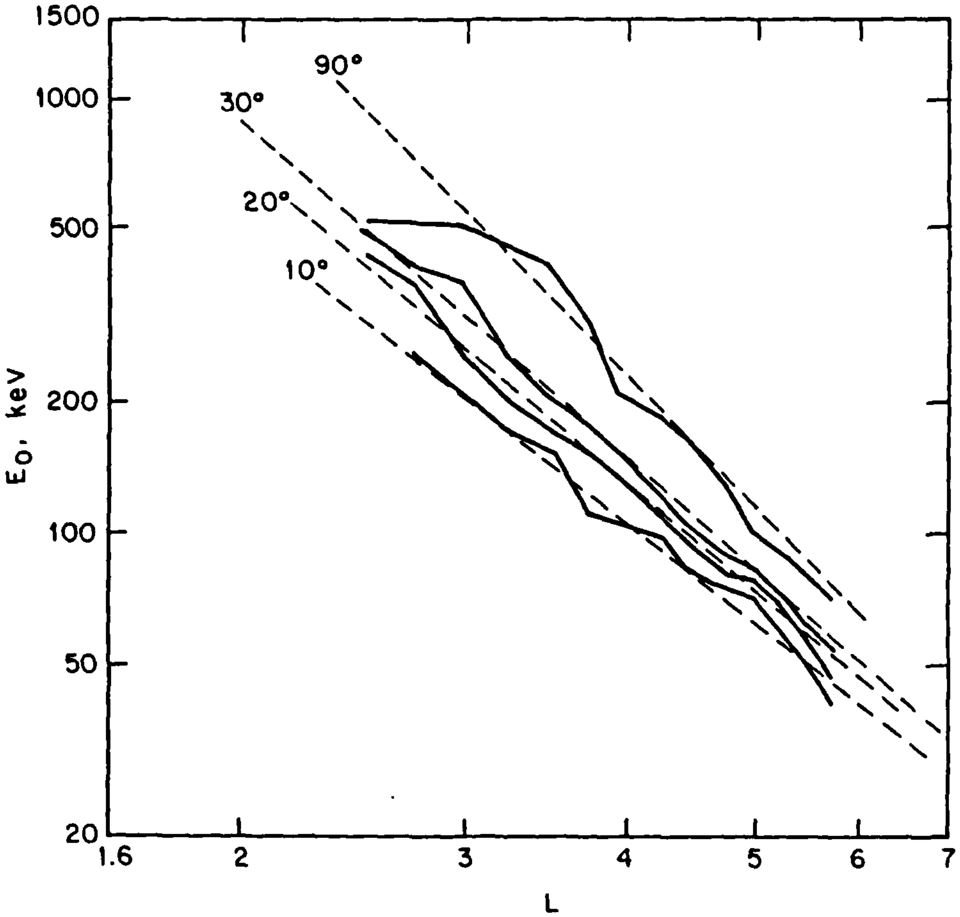

Figure 2. Empirical e-folding energies (solid curves) of observed proton spectra (Davis and Williamson, 1963) at equatorial pitch angles consistent with (5) and mapping to α_0 = 10°, 20°, 30°, and 90° (respectively) at L = 7; expected variation (dashed curves) of energy with L for individual protons having constant M and J, for selected values of energy and equatorial pitch angle at L = 7 (Dungey et al., 1965).

E_0^* and L^* suggest that the adiabatic and non-adiabatic energization processes operative in the plasma sheet must be able to provide a proton temperature ~ 12 keV at $L \sim 10$, at least for the high-energy tail of the proton distribution there. The appearance of a boundary at $L^* \sim 10$ between closed and open drift shells is quite plausible for the (high) proton energies of interest.

The large anisotropy inherent in (8) and (9) for $L \ll L^*$ offers a likely explanation for the occurrence of pitch-angle distributions such as those illustrated in Figure 3 (Blake et al., 1973). The proton anisotropy at $L = 1.85$ is an increasing function of energy, as would be expected from the presence of an exponential factor in (9). The alpha-particle anisotropy is even larger than the proton anisotropy at the same energy/nucleon, as would be expected if the alpha-particle temperature were somewhat less than four times the proton temperature at the outer boundary of trapped radiation, i.e., at $L = L^*$. Such a boundary condition is not really implausible in view of the adiabatic and non-adiabatic plasma-sheet processes (see above) that must intervene before a solar-wind ion enters the radiation belts. Moreover, there is no expectation that plasma-sheet ions of ionospheric (rather than solar-wind) origin would even have entered the plasma sheet with kinetic energy proportional to ionic mass.

It is not really accurate to say that the observed hardening of the proton spectrum with decreasing L in Figure 2 is a consequence of Liouville's theorem. It is not true, for example, that $\bar{f}(M, J, L) = \bar{f}(M, J, L^*)$. This can be seen from (6). On the other hand, Liouville's theorem is not violated by the radial-diffusion process. The distinction that must be made here is that Liouville's theorem requires the

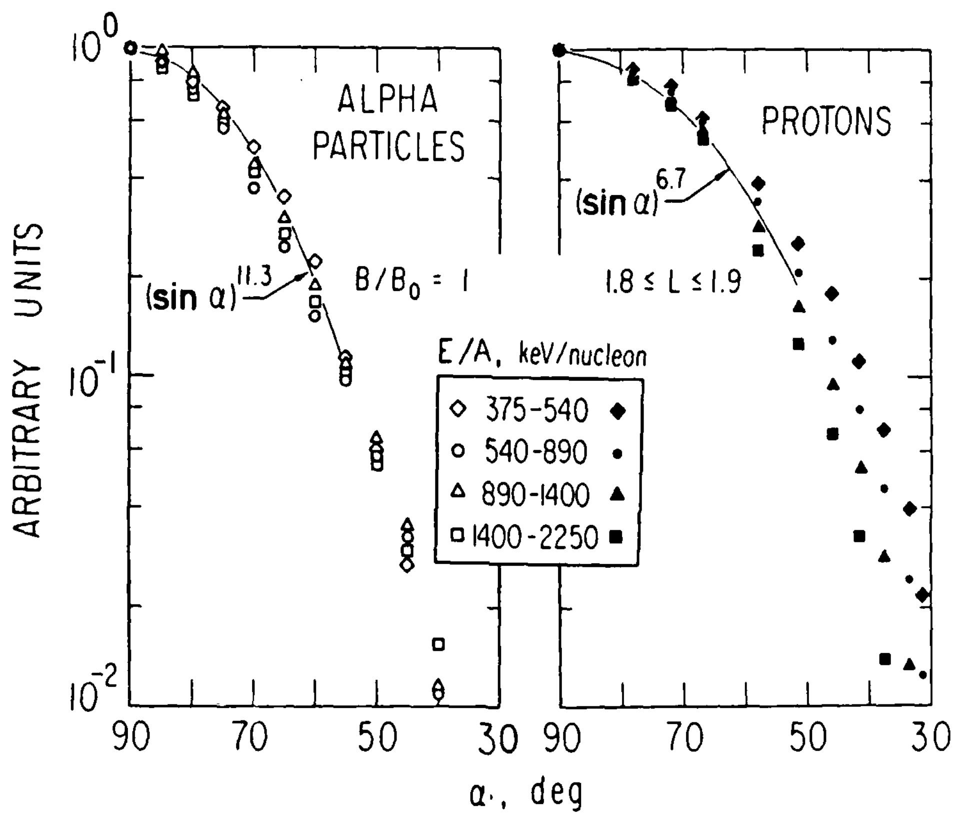

Figure 3. Equatorial pitch-angle distributions of alpha particles (left panel) and protons (right panel) in the same four energy/nucleon passbands, as observed on spacecraft OV1-19 (Blake et al., 1973). Statistical error bars (not shown) would be about as large as the data-point symbols themselves.

conservation of $f(M, J, L, \varphi)$ but not the conservation of $\bar{f}(M, J, L)$; the entity that diffuses is the longitude-averaged (i.e., φ-averaged or drift-averaged) phase-space density, whereas the longitude-dependent phase-space density remains conserved. The dilemma confronting the space researcher is that $f(M, J, L, \varphi)$ is difficult to observe with high resolution because particles that differ even slightly in M or J have different drift periods and thus become separated rather quickly in longitude (φ). The primary observable manifestation of drift-phase organization in the phase-space density is the drift-echo phenomenon, and even this is discernible for only a few drift periods following an impulsive change in the magnetospheric $\underset{\sim}{B}$ field (e.g., Lanzerotti et al., 1971). Nevertheless, the permanent dispersal of particles in L by such a Hamiltonian process definitely implies their permanent organization in φ for each M and J in the distribution, and such drift-phase organization really is a consequence of Liouville's theorem since the drift phase is canonically conjugate to the action variable $(q/c)\Phi$ for a particle of charge q and third invariant Φ (Roberts, 1967; Schulz and Lanzerotti, 1974).

The normalization constants 7×10^{-9} day^{-1} and 1×10^{-10} day^{-1} that appear in (3a) and (3b), respectively, are those obtained by Croley et al. (1976) from a variational analysis of inner-zone proton data. It follows that $D_{LL}^{(m)} \geq D_{LL}^{(e)}$ if $M/Z \geq 120$ MeV/gauss (i.e., if $E \geq 40\, Z/L^3$ MeV) for ions that mirror on the equator (i.e., for $y = 1$). For $y \ll 1$ one finds $D_{LL}^{(m)} \geq D_{LL}^{(e)}$ only if $E \geq 180\, Z/L^3$ MeV. These results imply, for example, that ring-current ions (for which $E \approx 10$-100 keV) are transported mainly by electrostatic impulses (unsteady magnetospheric convection). Among ions for which $E \geq 1.2\, Z/L$ MeV [so that $(\Omega_3\tau)^2 \geq 40$ in (3b)], the diffusion coefficient $D_{LL}^{(e)}$ scales as $(Z/A)^2$ at fixed K^2 $(= J^2/8m_0M)$ and fixed M/A (first invariant per nucleon). This is the scaling to which Cornwall (1972) appealed in his argument for preferential access of H^+ $(Z/A = 1)$ over He^{++} $(Z/A = 1/2)$ and He^+ $(Z/A = 1/4)$ to the inner part of the magnetosphere, e.g., to $L \leq 3$. Conversely, among ions for which $E \leq 30\,Z/L$ keV [so that $(\Omega_3\tau)^2 \leq 1/40$ in (3b)] the diffusion coefficient becomes independent of M, J, Z, and A. In this case one obtains $D_{LL}^{(e)} \approx 8 \times 10^{-5}(\tau/20 \text{ min})^2 L^6 \text{ day}^{-1}$. Ring-current ions occupy the middle range $(0.1 \leq \Omega_3\tau \leq 10)$ of the parameter $\Omega_3\tau$ between these two extremes and thus defy efforts to simplify the expression for $D_{LL}^{(e)}$. Perhaps this is not an accident, inasmuch as the decay time τ of an impulse in the convection electric field should naturally reflect the time scale of the underlying particle motion whereby the convection electric field might be discharged.

Although analyses based on radial diffusion alone are quite instructive, additional transport processes must be invoked if one is to account for further details of the prevailing ionic phase-space distributions. For example, one may ask where (in L) the radiation intensity peaks at a given energy E. Differentiation of (6) with respect to L with (9a) inserted for $\bar{f}(M, J, L^*)$ yields $(\partial\bar{f}/\partial L)_E = 0$ under the condition

$$\frac{7(L_1/L)^7}{1 - (L_1/L)^7} = - \left[\ell + 1 + \frac{E}{E_0} \right] \frac{d \ln E_0}{d \ln L} \qquad (10)$$

If the boundary spectrum is assumed to be Maxwellian (so that $\ell = -1$), then for particles mirroring at the equator one obtains

$$L \approx [(7/3)(L^*/L_1)^3(E_0^*/E)]^{1/10}[1 - (L_1/L)^7]^{1/10} L_1 \qquad (11)$$

as the location of peak radiation intensity, since $E_0 = (L^*/L)^3 E_0^*$ for $y^* = 1$ in (5) and (9b). The factor $[1 - (L_1/L)^7]^{1/10}$ varies only from 0.99 to 1.00 as L varies from about 1.6 to infinity, and so the expectation based on (6) is that the location in L of maximum radiation intensity should vary as $E^{-1/10}$. Observations compiled by Fritz and Spjeldvik (1981) show clearly that the actual variation closely resembles $E^{-3/16}$, as had been predicted by Tverskoy (1965), and not $E^{-1/10}$ for $1.5 \le L \le 4.5$; moreover, the peak alpha-particle intensity occurs about 1.3 times as far from the geocenter as the peak intensity for protons of the same energy, and this too had been predicted by Tverskoy (1965). The essential process considered by Tverskoy (1965) in addition to radial diffusion was Coulomb drag, which formally causes a particle's M to decrease while K^2 ($= J^2/8m_0M$) remains constant, i.e., causes a particle's energy to decrease while the equatorial pitch angle remains constant. In order to take advantage of the fact the K is conserved by both processes (radial diffusion and Coulomb drag) it is usual to transform from the canonical variables (M, J, Φ) to the "new" variables (M, K, L). The Jacobian of this transformation has an absolute value given by

$$|\partial(M, J, \Phi)/\partial(M, K, L)| = (8m_0M)^{1/2}(2\pi\mu/L^2 a) \qquad (12)$$

and so the appropriate Fokker-Planck equation is of the form

$$\frac{\partial \bar{f}}{\partial t} + M^{-1/2}\frac{\partial}{\partial M}\left[M^{1/2}\left(\frac{dM}{dt}\right)_\nu \bar{f}\right]_{K,L} = L^2\frac{\partial}{\partial L}\left[\frac{D_{LL}}{L^2}\frac{\partial \bar{f}}{\partial L}\right]_{M,K} \qquad (13)$$

Here the transport coefficient $(dM/dt)_\nu$ is given by

$$(dM/dt)_\nu = M^{-1/2}(4\pi Z^2 q_e^4/m_e)(L^9 a^9 y^6 m_0/2\mu^3)^{1/2}$$
$$\times \langle \sum_i N_i Z_i [\gamma^2 - 1 - \gamma^2 \ln(4\mu m_e M/m_0 y^2 L^3 a^3 I_i)]$$
$$+ N_e[\gamma^2 - 1 - \gamma^2 \ln(\lambda_D m_e v/\hbar)] \rangle \qquad (14)$$

where $\gamma^2 = 1 + (2\mu M/m_0 c^2 y^2 L^3 a^3) = [1 - (v/c)^2]^{-1}$, and where the subscript ν denotes "frictional drag" as distinguished from other processes. Other symbols appearing in (14) include the charge q_e and rest mass m_e of an electron, the densities N_e of free electrons and N_i of neutral atoms or molecules of atomic number Z_i, the mean energy I_i required for atomic excitation or ionization, the Debye length λ_D, and the symbol $\hbar$ (Planck's constant divided by 2π). Thus, for energetic ions of the same M/A and $K^{\sim}$, the transport coefficient $(dM/dt)_\nu$ scales as $A^{-1/2}Z^2$. This means that the ratio M/A decreases at a rate proportional to $A^{-3/2}Z^2$ for an ion of charge state Z and atomic-mass number A, i.e., as 64:32:8:1 for $H^+ : He^{++} : He^+ : O^+$.

The scaling of $(dM/dt)_\nu$ with L naturally depends on the models adopted for atmospheric and plasmaspheric density distributions. Tverskoy (1965) described Coulomb drag in terms of a "lifetime" proportional to $E^{3/2}Z^{-2}A^{-1/2}\langle N_e \rangle^{-1}$ throughout the radiation belts, and thereby replaced the basic transport equation with an equation that would yield some simple results in closed form. The model atmosphere and plasmasphere described by Farley and Walt (1971) yielded a "lifetime" $(-d \ln E/dt)^{-1}$ roughly proportional to $L^{-1.3}$ for protons of specified M and vanishing K in the interval $1.6 \le L \le 2.5$ (and presumably for higher L values within the plasmasphere). This would correspond to a transport coefficient $(dM/dt)_\nu$ proportional to $L^{1.3}$ over a region in which hydrogen atoms and free electrons are the dominant agents of Coulomb drag. The ion energy lost to neutral atoms and molecules in (14) results in elevation of bound electrons to higher energy levels (excitation) or to the spectral continuum (ionization). The energy lost to free electrons (via the term containing N_e and λ_D) results in the Cherenkov emission of Langmuir waves. The logarithms in (14) are typically quite large (≥ 10) for radiation-belt ions.

The angle brackets in (14) denote the guiding-center average over the bounce and drift motion. Evaluation of the bounce average typically entails a numerical computation, but a few analytical estimates are available in closed form. Cornwall (1975) suggests that the bounce-averaged atomic-hydrogen density scales as y^{-1} times the equatorial density. Smith et al. (1976) suggest that the bounce-averaged density of hydrogen atoms scales as $\sec^{3.5}\lambda_m$, where λ_m is the mirror latitude. Schulz (1977) has described a method of calculating, for any positive integer n, the bounce-averaged value of $(B/B_0)^n$ along a dipolar field line on which B_0 ($\equiv \mu/L^3a^3$) is the equatorial value of the magnetic-field intensity B. For example, the case n = 1 yields the result

$$\langle (B/B_0) \rangle \approx (1.9191\, y^{-5/4} - 1.1786\, y^{-1}) \div (1.3802 - 0.6397\, y^{3/4}) \qquad (15)$$

which might be relevant to a model in which N_e is proportional to B along a plasmaspheric field line. For magnetic drift shells that intersect the model plasmapause, account must be taken of the relative amounts of time spent inside and outside the plasmasphere in evaluating the azimuthal (drift) averages of N_e and $N_e \ln(\lambda_D m_e v/\hbar)$ for use in (14). The dependence of λ_D on N_e is not a severe complication here, since the argument of the logarithm is typically very large, i.e., since the dependence of the logarithm on N_e is typically quite weak. Thus, it should be sufficient to define $\bar{\lambda}_D$ as the value of λ_D at $B/B_0 = \langle(B/B_0)\rangle$ and to approximate the bounce average of $N_e \ln(\lambda_D m_e v/\hbar)$ by the quantity $\langle N_e\rangle \ln(\bar{\lambda}_D m_e v/\hbar)$.

LOSS PROCESSES

If Coulomb drag is considered a transport process, as seems appropriate, then the main loss mechanism for ring-current and radiation-belt ions appears to be charge exchange. In its simplest manifestation charge exchange causes a singly charged ion to become neutral and thus escape magnetic confinement by the earth's field. This process is described formally by adding a term $-\bar{f}/\tau_q$ to the right-hand side of (13). The charge-exchange lifetime τ_q is given by $\tau_q^{-1} = \langle N_H \rangle \sigma v$, where N_H is the density of atomic hydrogen in the exosphere, v is the speed of the ion [as in (14) above], and σ is the ion's cross section for charge exchange with atomic hydrogen. The structure of (13) makes it desirable to specify τ_q as a function of K, L, and M. Results for H^+ and He^+ ions at K = 0 in a model atmosphere of atomic hydrogen are shown in Figure 4 (Cornwall, 1971). Lifetimes for H^+ ions are thus typically shorter than for He^+ ions of the same K and M/A. Both $\langle N_H \rangle$ and σv vary with L so as to produce the results shown in Figure 4; the ion energy (on which σv depends quite strongly) varies as L^{-3} for K = 0. The products σv are shown as a function of ion energy for various ions in Figure 5 (Tinsley, 1976; Cornwall and Schulz, 1979). The cross sections σ correspond to neutralization for H^+, C^+, N^+, and O^+. Those for helium ions describe this as well as other possibilities, viz., $He^{++} \longrightarrow He^+$ and $He^+ \longrightarrow He^0$ (dashed curves) and $He^+ \longrightarrow He^{++}$ (shown by the +++-- curve at E ≥ 150 keV in Figure 5).

The possibility of two or more charge states for an ionic species

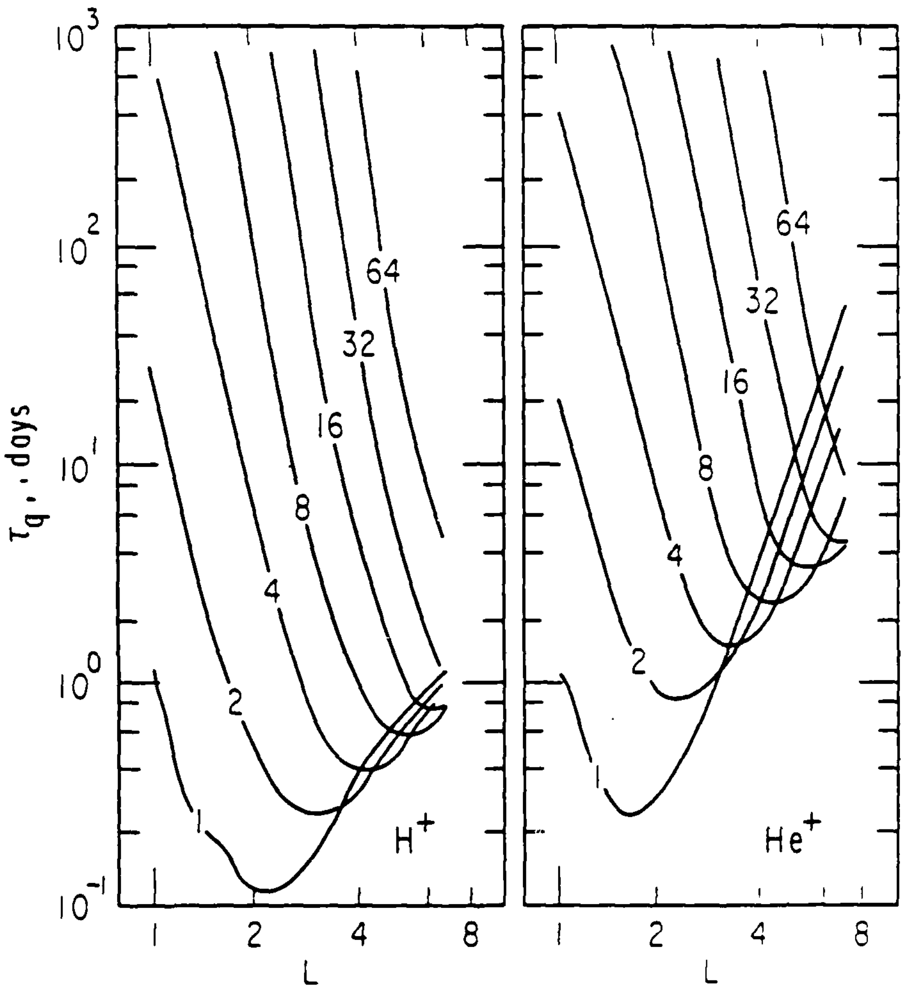

Figure 4. Charge-exchange lifetimes against neutralization in model atomic-hydrogen atmosphere for equatorially mirroring (K = 0) protons (H^+, left panel) and helium ions (He^+, right panel) at selected values of M/A, measured in MeV/gauss-nucleon (Cornwall, 1971; Schulz and Lanzerotti, 1974).

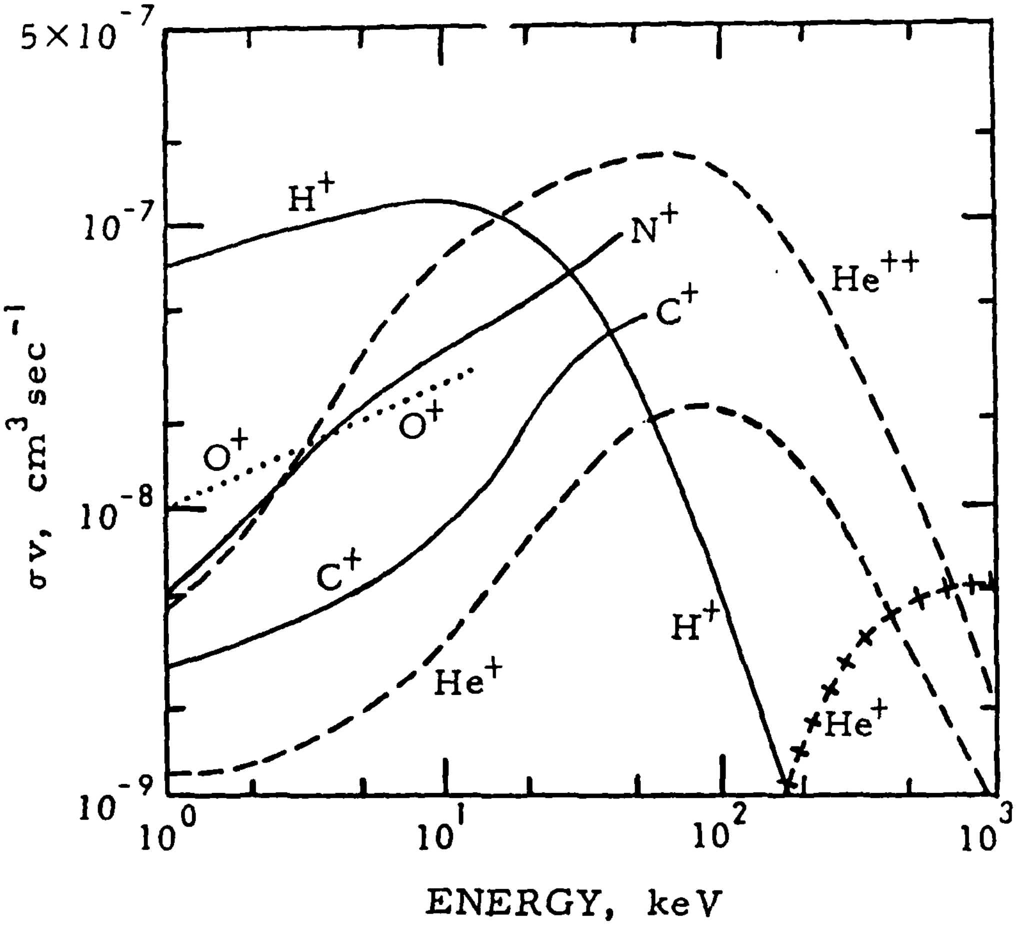

Figure 5. Products of ion velocity and cross section for charge exchange in atmosphere of atomic hydrogen (Cornwall and Schulz, 1979), based on compilations summarized by Cornwall (1971) and Tinsley (1976). Cross sections correspond to neutralization for H^+, C^+, N^+ (solid curves), O^+ (dotted curve), and He^+ (dashed curve); to reduction of charge state (He^{++} to He^+) for He^{++} (dashed curve); and to increase of charge state (He^+ to He^{++}) for He^+ (curve of "plus" signs at lower right).

requires that coupled transport equations be introduced. For helium ions the relevant phase-space densities are $\bar{f}_1$ and $\bar{f}_2$, the phase-space densities of He^+ and He^{++}, respectively. The coupled transport equations are of the form

$$(\partial \bar{f}_1/\partial t) + \mathbf{C}_1 \bar{f}_1 = \mathbf{D}_1 \bar{f}_1 - (\bar{f}_1/\tau_{10}) - (\bar{f}_1/\tau_{12}) + (\bar{f}_2/\tau_{21}) \quad (16a)$$

and

$$(\partial \bar{f}_2/\partial t) + \mathbf{C}_2 \bar{f}_2 = \mathbf{D}_2 \bar{f}_2 - (\bar{f}_2/\tau_{21}) + (\bar{f}_1/\tau_{12}) \quad (16b)$$

where $\mathbf{C}$ and $\mathbf{D}$ are the Coulomb-drag and radial-diffusion operators that appear in (13), while τ_{ij} denotes the lifetime against transition from charge state i to charge state j. Thus, the lifetime τ_{10} in (16a) is identical with the τ_q that appears in the helium (He^+) panel of Figure 4. Spjeldvik and Fritz (1978) have generalized (16) for application to the charge states of oxygen, which are quite numerous but present no difficulty in principle.

Strictly speaking, the source terms $\bar{f}_2/\tau_{21}$ in (16a) and $\bar{f}_1/\tau_{12}$ in (16b) should be replaced by nonlocal source terms in which $\bar{f}_i(L')$ is averaged (with appropriate weighting) over a narrow interval $|L' - L| < \Delta L$ about the L value of interest. Such a refinement of (16) would acknowledge the fact that a change in charge state inherently displaces the guiding center of a geomagnetically trapped ion by instantaneously altering the gyro-radius. For ions of initial charge state Z and initial gyro-radius ρ at the equator, the maximum change in L value from transition to charge state $Z \pm 1$ would be of order $\Delta L = (\rho/a)(Z \pm 1)^{-1}$, where a is the radius of the earth. Such considerations would normally be unimportant, since the adiabatic description of charged-particle motion (upon which ring-current and radiation-belt theory is founded) requires $\rho/La \ll y \leq 1$ for its validity. However, charge exchange from state i to state j does not remove an ion from the radiation belt unless $j = 0$, nor does charge exchange entail significant pitch-angle diffusion of the trapped ion. Thus, the same ion can experience charge exchange between the same pair of states many times during its residence in the magnetosphere, and the effect of introducing nonlocal source terms to account for this in the equation for $\partial \bar{f}_j/\partial t$ would simulate augmentation of the radial-diffusion coefficient by an amount $\Delta D_{LL}^{(j)} \sim \langle (\rho/a)^2 (Z \pm 1)^{-2} (\tau_{ij} + \tau_{ji})^{-1} \rangle$. Such an augmentation might (or might not) be quantitatively significant.

Transition from charge state $Z = 1$ to charge state $Z - 1$ ($= 0$) displaces the guiding center to infinity and thus violates the adiabatic description of charged-particle motion. Such an event (i.e., neutralization) removes a fast ion from the ring current but creates a fast neutral whose direction of motion depends on a random variable, viz., the ion's gyrophase at the instant of neutralization. It takes time (~ 20 sec) for the fast neutral to escape from the magnetosphere, and during this time the fast neutral can be "stripped" of one or more electrons so as to re-appear as a born-again ion elsewhere in the magnetosphere. Such reconversion of a fast neutral into an ion is unlikely to occur unless the trajectory of the neutral carries it through the dense atmosphere immediately surrounding the earth. However, Mizera and Blake (1973) have indeed reported evidence of this process in the form of a partial belt of born-again ions of ring-current energy at $L \leq 1.1$ near the magnetic equator. Observation of the effect is facilitated by the eccentricity (~ 440 km) of the drift shells relative to the earth, since ions reborn at a 100-km altitude just west of the South Atlantic anomaly would gradient-drift to an altitude ≥ 900 km in about half a drift period.

Despite the above-described implications for ion transport, charge exchange is primarily a loss mechanism for ring-current ions. It follows from Figure 5 that H^+ is lost more rapidly than O^+ or He^+ at ion energies ≤ 50 keV. Lyons and Evans (1976) have emphasized the importance of charge exchange in this context for the ion-composition of the recovery-phase ring current. Their conclusion was that the ring-current ions observed on the S^3 satellite (also known as Explorer 45) decayed much too slowly in intensity (and developed pitch-angle anisotropy much too slowly) to have been H^+ ions. This was a disturbing suggestion at the time, since the predominance of H^+ among the ions of the ring current had by then become an article of faith. The argument of Lyons and Evans (1976) was not air-tight, since it ignored (a) the likely possibility that radial diffusion would continue during recovery phase and (b) the adiabatic energization that ring-current particles would experience through the azimuthal electric field induced by decay of the ring current itself. However, the work of Lyons and Evans (1976) alerted space researchers to the likelihood (now generally accepted as fact) that ions heavier than H^+ are major constituents of the ring current at least part of the time. The abundance of O^+ in auroral ion beams (see above under "Access") makes this conclusion seem less surprising in retrospect.

The creation of pitch-angle anisotropy in ring-current ion distributions is an expected consequence of charge exchange, since the bounce-averaged density $\langle N_H \rangle$ of atomic hydrogen in the atmosphere should be an increasing function of mirror latitude λ_m at each L value. It was noted above in connection with Coulomb drag that several functional forms have been proposed for scaling $\langle N_H \rangle$ with equatorial pitch angle ($\alpha_0 \equiv \sin^{-1} y$) or mirror latitude, e.g., y^{-1} by Cornwall (1975) and $\sec^{3.5}\lambda_m$ by Smith et al. (1976). The similarity of these two scalings is revealed by the relationship

$$y = (1 + 3\sin^2\lambda_m)^{-1/4} \cos^3\lambda_m \tag{17}$$

that holds between y and λ_m in a dipolar $\mathbf{B}$ field. If processes other than charge exchange are neglected for simplicity, then an initially isotropic phase-space density $\bar{f}(E, L; 0)$ evolves into the distribution

$$\bar{f}(E, L, y; t) = \exp(-\langle N_H \rangle \sigma v t)\, \bar{f}(E, L; 0) \tag{18}$$

which is anisotropic for times $t > 0$ by virtue of the dependence of $\langle N_H \rangle$ on y. Cornwall (1977) realized that such anisotropy among ring-current ions could lead to instability in electromagnetic ion-cyclotron wave modes. He was able to calculate the growth rate in closed form as a function of wave frequency ($\omega/2\pi$) and elapsed time (t) under the simplifying (but admittedly unrealistic) approximation that $\langle N_H \rangle$ scales as y^{-2} rather than as y^{-1} or as $\sec^{3.5}\lambda_m$. His results are nevertheless revealing and qualitatively consistent with expectation. The growth rate is initially negative at all frequencies $\omega/2\pi > 0$ because the pitch-angle distribution is isotropic, as can be seen from (18) for $t = 0$. Thereafter the distribution becomes anisotropic, and so the growth rate becomes positive for a band of frequencies $\omega/2\pi$ between zero and some fraction of the ion (in this case, proton) gyrofrequency $\Omega/2\pi$. Experience shows that the fraction should be approximately $A/(A + 1)$, where A is the anisotropy of the distribution. The fraction is exactly $A/(A + 1)$, for example, if the pitch-angle distribution has the form $\sin^{2A}\alpha_0$ or corresponds to a bi-Maxwellian distribution for which the temperature ratio is given by $T_\perp/T_\parallel = A + 1$. Of greater interest than the local growth rate Γ is the path-integrated gain

$$G = \exp \oint (2\Gamma/v_g)\, ds \tag{19}$$

where v_g is the group velocity and ds is the element of arc length along the ray path (usually taken to be a field line in order to simplify the task). It is common in magnetospheric physics to assume that $G \approx \exp(4\Gamma_0 La/v_g)$, where Γ_0 is the equatorial growth rate, in order to simplify the task further. However, Liemohn (1967) has demonstrated interesting effects that are overlooked if the path integral is not evaluated more carefully. In any event, the gain G has a maximum with respect to $\omega/2\pi$, and the maximum gain thus determined is not monotonic as a function of t. Indeed, the value of $G_{max}(t)$ is larger than unity for $0 < t < \infty$ but equal to unity for $t = 0$ (when the anisotropy A vanishes) and for $t = \infty$ (when $\bar{I}$ itself vanishes by virtue of charge exchange). Thus, if $G_{max}(t)$ increases monotonically for $t < t^*$ and decreases monotonically for $t > t^*$, the remaining question is whether $G_{max}(t^*)$ is sufficiently large for instability to occur at all. The usual criterion for instability is $G_{max}(t^*) \geq 400$, i.e., a single-pass gain of at least 20 in wave intensity (or 13 dB in decibel notation) from one foot of the field line to the other, since it is presumed that only about

5% of the wave intensity that reaches either foot of the field line will be reflected back into the magnetosphere. The integral in (19) refers to propagation once in <u>each direction</u> along the field line and so must compensate for two such 5% reflections. If the instability occurs, i.e., if $G_{max}(t^*) \geq 400$, then the resulting ion-cyclotron waves will enhance the loss rate of ring-current ions by causing pitch-angle diffusion into the loss cone. Such diffusion will naturally reduce the anisotropy of the pitch-angle distribution and thus limit the instability, but it will also enhance the effectiveness of charge exchange as a loss mechanism for ring-current ions by increasing the mirror latitude (and hence the magnitude of $\langle N_H \rangle$) for the average ion trapped in the flux tube.

It follows from the above considerations that the probability of having pitch-angle diffusion among ring-current ions increases with the strength of the magnetic storm, i.e., with the intensity of the ring current itself. Recovery from a weak storm might be achieved through charge exchange alone, whereas recovery from a major storm might involve both charge exchange and pitch-angle diffusion (each serving to enhance the effectiveness of the other). The foregoing scenario is incomplete in at least three respects. First, the initial pitch-angle distribution is not likely to be isotropic, since the radial transport that populates the ring current from the plasma sheet creates pitch-angle anisotropy even when charge exchange and Coulomb drag are neglected. This was illustrated above (but for higher-energy ions) in Figure 2 and in equations (5)-(9). Thus, wave growth occurs even at $t = 0$, although $G_{max}(0)$ is likely to be smaller than $G_{max}(t^*)$ for some $t^* > 0$.

Secondly, the ring current is likely to be populated by two or more ionic species simultaneously. Perhaps H^+ is the major ion initially, i.e., during the main phase of a magnetic storm, with He^{++}, He^+, and O^+ present as (relatively) minor constituents. In this case the electromagnetic mode structure is interrupted by stop-bands above the gyrofrequencies of the heavier ions, and the hydrogen-cyclotron instability is partially suppressed in favor of enhanced excitation at frequencies below the gyrofrequencies of the helium and oxygen ions (e.g., Cornwall and Schulz, 1971; Chiu et al., 1980). Ions having gyrofrequency $\Omega/2\pi$ can experience pitch-angle diffusion through a Doppler-shifted cyclotron resonance with a frequency $\omega/2\pi$ represented in the wave spectrum if the ion energy is sufficient to satisfy the resonance condition

$$kv = (\omega - \Omega)\sec\alpha > 0 \tag{20}$$

where $2\pi/k$ is the wavelength that corresponds to the wave frequency $\omega/2\pi$. Resonance for $\Omega > \omega$ requires $\cos\alpha < 0$, i.e., particle and wave traveling in opposite directions along the field line. Resonance for $\Omega < \omega$ requires particle and wave to travel in the same direction along the field line so that $\cos\alpha > 0$. As H^+ ceases to be the major ion, the stop bands above the helium gyrofrequencies grow larger and suppress the hydrogen-cyclotron instability altogether (e.g., Chiu et al., 1980), but instability can still occur in the He^{++}, He^+, and O^+ passbands.

Finally, the foregoing scenario is incomplete in that electrostatic waves have been neglected. The usual rule-of-thumb in magnetospheric physics is that electromagnetic waves predominate inside the plasmasphere and electrostatic waves outside. Electromagnetic ion-cyclotron waves are easier to analyze because the cold-plasma dispersion relation can be perturbed in order to extract hot-plasma effects such as wave growth. This does not usually work for electrostatic ion-cyclotron waves, since in this case the cold-plasma dispersion relation does not provide a good approximation of the phase velocity. The usual spectrum of electrostatic ion-cyclotron turbulence is characterized by excitation bands located about midway between harmonics of the ion gyrofrequency, and the usual source of free energy is anisotropy (e.g., $T_{\perp} > T_{\parallel}$) in the pitch-angle distribution. Interesting complications should ensue for the ring-current plasma outside the plasmasphere, since (for example) the $\frac{3}{2}$ harmonic of the H^+ gyrofrequency coincides with integer harmonics (the third, sixth, and twenty-fourth, respectively) of the He^{++}, He^+, and O^+ gyrofrequencies.

PERSPECTIVE

Implementation of the foregoing principles may impress the reader as an onerous task. It is natural to ask whether a simpler theory of magnetospheric ion composition is not already available. In a certain sense the theory of Tverskoy (1965) fills this request. Tverskoy (1965) argued that the maximum in radiation intensity at any energy E should be located at the L value for which the "time scales" associated with radial diffusion and Coulomb drag are equal. This would occur where $M^{-1}(dM/dt)_\nu = -L^{-2}D_{LL}$ in the language of the present work. Tverskoy (1965) neglected the Coulomb drag associated with neutral atoms and molecules in (14), as well as the radial diffusion associated with electrostatic impulses (unsteady convection) in (3). On this basis one would expect to find

$$(4\pi Z^2 q_e^4/m_e)(m_0/2E^3)^{1/2}\langle N_e\rangle \ln(\bar{\lambda}_D m_e v/\hbar)$$
$$\approx 8.1 \times 10^{-14} L^8 [Q(y)/180\,D(y)]^2 \ \sec^{-1} \qquad (21)$$

at the L value of maximum radiation intensity for specified E, Z, A, and y, since the argument of the logarithm in (21) is very large ($\geq 10^7$) and the value of γ is of order unity in most situations of interest. Solution of (21) for L yields

$$L_{max} \approx 3.00\langle(N_e/10^3 \ \mathrm{cm}^{-3})\rangle^{1/8} [(1/16)\ln(\bar{\lambda}_D m_e v/\hbar)]^{1/8}$$
$$\times [180\,D(y)/Q(y)]^{1/4} Z^{1/4} A^{1/16} (E/1 \ \mathrm{MeV})^{-3/16} \qquad (22)$$

as the location of peak radiation intensity. The normalization of N_e, E, and $\ln(\bar{\lambda}_D m_e v/\hbar)$ by typical values (10^3 cm^{-3}, 1 MeV, and 16, respectively) of these quantities makes the above expression for L_{max} easier to evaluate by inspection. It was on the basis of a similar expression that Tverskoy (1965) anticipated the $E^{-3/16}$ scaling law for L_{max} and the appearance of the alpha-particle ($Z = 2$, $A = 4$) maximum at 1.3 ($\approx 2^{3/8}$) times as high an L value as the intensity maximum for protons of the same energy.

The scaling laws deduced by Tverskoy (1965) are amply confirmed by the observations that Fritz and Spjeldvik (1981) have recently compiled. Even the leading factor 3.00 in (22), which is not arbitrary but derived from (21), seems appropriate in that (for $y = 1$) the 1-MeV proton maximum occurs at $L \sim 3$ and the 1-MeV alpha-particle maximum at $L \sim 4$ (actually at $L = 2.76$ and $L = 3.59$, respectively, upon more careful inspection). Such good agreement seems surprising in view of the fact that (21) and (22) have no fundamental basis but only heuristic justification. On closer examination, however, the agreement is less impressive since (for example) the $E^{-3/16}$ scaling law for L_{max} follows from (21) only if $\langle N_e \rangle$ is independent of L. The more likely alternative scaling in which $\langle N_e \rangle$ is proportional to L^{-4} would have yielded $L_{max} \propto Z^{1/6} A^{1/24} E^{-1/8}$, in substantially poorer agreement with the data compiled by Fritz and Spjeldvik (1981). Furthermore, the "alpha-particle" observations ordinarily refer to a mixture of $Z = 1$ and $Z = 2$ charge states rather than to $Z = 2$ alone, since the charge-exchange rates that prevail in the earth's magnetosphere make this mixture unavoidable (Cornwall, 1972). It is not the purpose of this discussion to quibble over details of significant work that is nearly twenty years old, but rather to illustrate how much more sophisticated the treatment of radiation-belt dynamics has become over the intervening period. Just as the heuristic estimates by Tverskoy (1965) satisfied the needs of an earlier decade of space research, so the work of Spjeldvik and Fritz (1978) exemplifies the more fundamental and quantitative approach that is demanded today.

The present work is not intended as a compendium of "who has done what" in the field of magnetospheric ion composition, but rather as an introduction to the principles upon which the modern dynamical theory of geomagnetically trapped ion populations is based. Review articles with extensive bibliographies on the subject have been prepared by Cornwall and Schulz (1979), Spjeldvik (1979), and Fritz and Spjeldvik (1981), among others. The present assignment seemed to require a different approach, and the foregoing pages represent the outcome of this endeavor.

REFERENCES

Axford, W. I., On the origin of radiation belt and auroral primary ions, in Particles and Fields in the Magnetosphere, edited by B. M. McCormac, pp. 46-59, Reidel, Dordrecht, 1970.

Blake, J. B., J. F. Fennell, M. Schulz, and G. A. Paulikas, Geomagnetically trapped alpha particles, 2, The inner zone, J. Geophys. Res. 78, 5498-5506, 1973.

Burton, R. K., R. L. McPherron, and C. T. Russell, An empirical relationship between interplanetary conditions and D_{st}, J. Geophys. Res. 80, 4204-4214, 1975.

Chen, A. J., and D. P. Stern, Adiabatic Hamiltonian of charged particle motion in a dipole field, J. Geophys. Res. 80, 690-693, 1975.

Chiu, Y. T., J. M. Cornwall, J. G. Luhmann, and M. Schulz, Argon-ion contamination of the plasmasphere, in Space Systems and Their Interactions with Earth's Space Environment, edited by H. B. Garrett and C. P. Pike, pp. 118-147, AIAA, New York, 1980.

Cornwall, J. M., Transport and loss processes for magnetospheric helium, J. Geophys. Res. 76, 264-267, 1971.

Cornwall, J. M., Radial diffusion of ionized helium and protons: A probe for magnetospheric dynamics, J. Geophys. Res. 77, 1756-1770, 1972.

Cornwall, J. M., Moment transport equations for wave-particle interactions in the magnetosphere, J. Geophys. Res. 80, 4635-4642, 1975.

Cornwall, J. M., On the role of charge exchange in generating unstable waves in the ring current, J. Geophys. Res. 82, 1188-1196, 1977.

Cornwall, J. M., and M. Schulz, Electromagnetic ion-cyclotron instabilities in multicomponent magnetospheric plasmas, J. Geophys. Res. 76, 7791-7796, 1971; Correction, J. Geophys. Res. 78, 6830, 1973.

Cornwall, J. M., and M. Schulz, Physics of heavy ions in the magnetosphere, in Solar System Plasma Physics, edited by L. J. Lanzerotti, C. F. Kennel, and E. N. Parker, vol. 3, pp. 165-210, North-Holland Publ. Co., Amsterdam, 1979.

Croley, D. R., Jr., M. Schulz, and J. B. Blake, Radial diffusion of inner-zone protons: Observations and variational analysis, J. Geophys. Res. 81, 585-594, 1976.

Davidson, G. T., An improved empirical description of the bounce motion of trapped particles, J. Geophys. Res. 81, 4029-4030, 1976.

Davis, L. R., and J. M. Williamson, Low-energy trapped protons, Space Res. 3, 365-375, 1963.

Dungey, J. W., W. N. Hess, and M. P. Nakada, Theoretical studies of protons in the outer radiation belt, Space Res. 5, 399-403, 1965.

Farley, T. A., and M. Walt, Source and loss processes of protons of the inner radiation belt, J. Geophys. Res. 76, 8223-8240, 1971.

Fritz, T. A., and W. N. Spjeldvik, Steady-state observations of geomagnetically trapped energetic heavy ions and their implications for theory, Planet. Space Sci. 29, 1169-1193, 1981.

Ghielmetti, A. G., R. G. Johnson, R. D. Sharp, and E. G. Shelley, The latitudinal, diurnal, and altitudinal distributions of upward flowing energetic ions of ionospheric origin, Geophys. Res. Lett. 5, 59-62, 1978.

Gloeckler, G., A versatile detector system to measure the charge states, mass compositions and energy spectra of interplanetary and magnetospheric ions (abstract), in 15th International Cosmic Ray Conference, Conference Papers, vol. 9, abst. T51, p. 207, Bulg. Acad. Sci., Sofia, Plovdiv meeting, August 1977.

Kennel, C. F., and F. V. Coroniti, Is Jupiter's magnetosphere like a pulsar's or earth's?, Space Sci. Rev. 17, 857-883, 1975.

Lanzerotti, L. J., C. G. Maclennan, and M. F. Robbins, Proton drift echoes in the magnetosphere, J. Geophys. Res. 76, 259-263, 1971.

Liemohn, H. B., Cyclotron-resonance amplification of VLF and ULF whistlers, J. Geophys. Res. 72, 39-55, 1967.

Lyons, L. R., and D. S. Evans, The inconsistency between proton charge exchange and the observed ring current decay, J. Geophys. Res. 81, 6197-6200, 1976.

Lyons, L. R., and T. W. Speiser, Evidence for current sheet acceleration in the geomagnetic tail, J. Geophys. Res. 87, 0000-0000, 1982.

Mizera, P. F., and J. B. Blake, Observations of ring current protons at low altitudes, J. Geophys. Res. 78, 1058-1062, 1973.

Paulikas, G. A., and J. B. Blake, Penetration of solar protons to synchronous altitude, J. Geophys. Res. 74, 2161-2168, 1969.

Roederer, J. G., Dynamics of Geomagnetically Trapped Radiation, pp. 66, 127, Springer, Heidelberg, 1970.

Schulz, M., Bounce-averaged synchrotron loss in a dipole field, J. Geophys. Res. 82, 2815-2818, 1977.

Schulz, M., and L. J. Lanzerotti, Particle Diffusion in the Radiation Belts, pp. 17-22, 44, 83-95, Springer, Heidelberg, 1974.

Sckopke, N., and G. Paschmann, The plasma mantle: A survey of magnetotail boundary observations, J. Atmos. Terr. Phys. 40, 261-278, 1978.

Smith, P. H., N. K. Bewtra, and R. A. Hoffman, Decay mechanism for ring current ions (abstract SM11), Eos, Trans. Am. Geophys. Union 57, 982, 1976.

Speiser, T. W., Particle trajectories in model current sheets, 1, Analytical solutions, J. Geophys. Res. 70, 4219-4226, 1965.

Spjeldvik, W. N., Expected charge states of energetic ions in the magnetosphere, Space Sci. Rev. 23, 499-538, 1979.

Spjeldvik, W. N., and T. A. Fritz, Theory for charge states of energetic oxygen ions in the earth's radiation belts, J. Geophys. Res. 83, 1583-1594, 1978.

Stern, D. P., and P. Palmadesso, Drift-free magnetic geometries in adiabatic motion, J. Geophys. Res. 80, 4244-4248, 1975.

Tinsley, B. A., Evidence that the recovery phase ring current consists of helium ions, J. Geophys. Res. 81, 6193-6196, 1976.

Tverskoy, B. A., Transport and acceleration of charged particles in the earth's magnetosphere, Geomagnetizm i Aeronomiya 7, 226-242, 1965; English translation: Geomagn. Aeron. 7, 177-189, 1965.

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